Interim Information Design

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Abstract

We study efficient and consumer-surplus maximizing information policies in a bilateral trade setting where the buyer is initially privately imperfectly informed about his willingness to pay. We identify a canonical class of demand functions that can be implemented by information disclosures that are targeted based on the buyer's initial private information. As an application we show that providing more information to the buyer can lead to higher market prices and a lower trade probability without affecting the consumer or producer surplus.

Introduction

We revisit the information design problem in a single-item bilateral trade environment where the private disclosures to the buyer can impact her willingness to pay. The overall policy of such disclosures determine the demand faced by the seller and shape prices, consumer surplus and profits. See for example Roesler and Szentes (2017).

We analyze information design from an *interim* perspective. The buyer begins with some initial, partial, private infomation about her value for the object. This allows for disclosures that are targeted based on the buyer's private information but constrains the overall information policy to be no less informative than the initial, partially, informed buyer. We ask how market outcomes, like the probability

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of trade, prices, and welfare, are affected by interim information disclosures. In particular, we study how the designer's information policy can shape an *interim demand* into a *market demand* which determines the price and welfare.

We characterize all possible consumer-producer surplus pairs for any given distribution of initial private information. A result central to our analysis is the identification of a new class of canonical demands that are sufficient to generate all possible efficient consumer-producer surplus pairs. These demands resemble the iso-elastic demands that play a key role in Roesler and Szentes (2017), but have gaps and atoms resulting from the constraint imposed by the buyer's initial private information. We show that any efficient equilibrium demand can be transformed into a demand in this family through some information policy without affecting the welfare or price. Moreover, any equilibrium demand, not necessarily efficient, can be transformed into a demand in this family with possibly greater consumer surplus.

Roesler and Szentes (2017) characterize all possible equilibrium consumer-producer surplus pairs in a framework in which the buyer is initially uninformed (or equivalently the seller and buyer are initially symmetrically informed). They show that consumer-surplus maximizing information policies are efficient and entail trade with probability 1.

As trade always occurs in an efficient equilibrium, the total consumer-producer surplus equals the buyer's prior expectation of the item's valuation, and the producer surplus equals the price charged. As a result, in Roesler and Szentes (2017), the market price completely determines the consumer-producer surplus pair for any efficient outcome.

In our model, there could be many different corresponding market prices for any given efficient equilibrium consumer-producer surplus pair. Trade can occur with probability less than 1 for efficient outcomes due to the buyer's initial private information, resulting in extra flexibility between equilibrium welfare and prices. When the designer provides targeted information to buyers with an initial (interim) expected valuation below the market price, by Bayes plausibility, there is a positive probability that these buyers have a posterior expected valuation below the market price and do not trade.

As an application we derive a comparative static relationship between prices and information. Fixing some efficient consumer-producer surplus pair, as the buyer becomes better informed prices necessarily increase. This highlights an important friction involved with information provision in such environments; a more informed buyer makes a better purchase decision, but also faces higher prices. Importantly, we show that more informed buyers can drive up prices without affecting the welfare.

Finally, we provide a necessary condition for an outcome to be buyer optimal and characterize the buyer optimal outcome with the highest price.

Related Literature

Roesler and Szentes (2017) initiated the study of buyer-optimal information design in a bilateral-trade context. Condorelli and Szentes (2025) analyzed a similar problem in which the distribution of buyer's valuations come from a covert and costless investment decision rather than information acquisition. Park (2025) adapted the Roesler and Szentes (2017) framework to a setting in which a collection of buyers cooperatively purchase a public good. All of these papers study the design problem at the *ex ante* stage before the buyer(s) have any private information. In independent and concurrent research Ennuschat (2025) looks at a variation of the interim problem we study. In Ennuschat (2025) the underlying willingness to pay can take a continuum of values and the methodological approach is correspondingly different.

Model

We study a bilateral trade model with a buyer (he) and a monopolist seller (she). The seller offers a single item, produced at no cost. The buyer's underlying value for the product is binary and represented by $\theta \in \{0, 1\}$. We normalize the seller's reservation value to 0, hence trade is efficient. The buyer has partial private information about his valuation, represented by his prior belief $\mu \in [0, 1]$. We refer to the buyer's private belief μ as his type. The buyer's type is his interim expected valuation for the item.

There is a commonly known distribution $F \in \Delta([0, 1])$ from which types are drawn. The distribution F is the interim demand that the seller faces. The designer announces an information policy, denoted by $\rho(\cdot) = (\rho_0(\cdot), \rho_1(\cdot))$, where for each μ

$$\rho_\theta(\mu) \in \Delta(\mathcal{M})$$

is the distribution over signals that the buyer observes conditional on the true value of the object being θ. Here $\rho(\mu)$ represents a statistical experiment about θ whose realization is in some arbitrary set $\mathcal{M}$ and is only observed by the type μ buyer.

A type μ buyer observes the realization of $\rho(\mu)$ and updates his prior expected valuation μ to a posterior expected valuation ν as per Bayes rule. As the buyer's expected valuation dictates his decision to trade, each experiment $\rho(\mu)$ can be identified with the distribution of posterior valuations it induces for type μ buyer. Let $\eta(\mu) \in \Delta([0,1])$ represent this distribution of posterior expected valuations. In particular, it is without loss to identify an information policy with a mapping from types to a distribution of posterior expected valuations, represented by $\eta : [0,1] \to \Delta([0,1])$.

In response to the announced information policy, the seller forms a posterior distribution (*market demand*) G over the buyer's posterior expected valuation and offers a price p to the buyer. For any given information policy with corresponding η the market demand G is such that for all $\nu \in [0,1]$

$$G(\nu) = \mathbf{E}_F\left[\ \eta(\mu)(\ [0,\ \nu]\ )\ \right]$$

In other words $G(\nu)$ is the total probability under F and η that the buyer obtains posterior expected valuation less than or equal to ν.

The designer can induce a distribution G using some information policy $\rho(\cdot)$ if and only if $G \succeq_{mps} F$ (see Elton and Hill (1992) theorem 4.1 for details). Where $G \succeq_{mps} F$ holds whenever for every real valued continuous convex function φ we have the following

$$\int_0^1 \varphi(s)dG(s) \geq \int_0^1 \varphi(s)dF(s)$$

By Theorem 3.A.1 in Shaked and Shanthikumar (2007), the above is equivalent to the following

$$\int_0^p G(s)ds \geq \int_0^p F(s)ds \ \text{ for all } \ p \in [0,1]$$

where the above holds with equality at $p = 1$.

We will refer to such a distribution G as a *feasible distribution* (or *feasible demand*). We will work with the induced *market demand* G instead of the underlying information policy.

Feasible Demands

As pointed out in the previous section, given a prior F, the set of feasible seller's beliefs about the distribution of buyer's valuations (type) is given by

$$\mathcal{G}(F) := \left\{ G : [0,1] \to [0,1] \mid G \succeq_{mps} F,\ G \text{ is a cdf} \right\}$$

In our interim setting, the buyer has initial private information and the designer can provide information about the value θ type-by-type. For each buyer type μ, there corresponds a distribution of a posterior expected valuations $\eta(\mu)$ resulting from the information provided by the designer. The market demand faced by the seller is the average across types μ of these type-contingent posterior valuations.

By contrast, when the buyer and seller are initially symmetrically uninformed as in Roesler and Szentes (2017), the set of feasible market demands is given by

$$\mathcal{G}_{\text{UI}}(F) := \left\{ G : [0,1] \to [0,1] \mid F \succeq_{mps} G,\ G \text{ is a cdf} \right\}$$

Here is a way to understand the difference. In both models F is the seller's prior distribution over the buyer's valuation. In Roesler and Szentes (2017) the buyer begins with no information so F is also the buyer's prior distribution over her own valuations and therefore with probability 1 her interim willingness to pay is $\mathbf{E}F$. And in Roesler and Szentes (2017), the most the buyer could learn with full information is the realization of F. Thus in Roesler and Szentes (2017), the set of market demands is the set of mean-preserving contractions of F. By contrast in our framework the buyer begins already knowing the realization of F and therefore its realization represents the *least* the buyer could learn. The set of market demands is the set of mean-preserving *spreads* of F.

We can visualize the difference between the two settings concisely using the integral of the cumulative distribution function (see Figure 1). In particular, for a cumulative distribution function G we define the following function

$$C_G(s) = \int_0^s G(\mu) d\mu$$

We will refer to such an integral as the *information functional* generated by the corresponding cumulative distribution function G. The following information functionals are of particular interest

$$C_F(s) = \int_0^s F(\mu) d\mu, \quad C_{\text{NI}}(s) = \int_0^s 1_{\{v \geq \mathbf{E}F\}} d\mu, \quad C_{\text{FI}}(s) = s\,(1 - \mathbf{E}F)$$

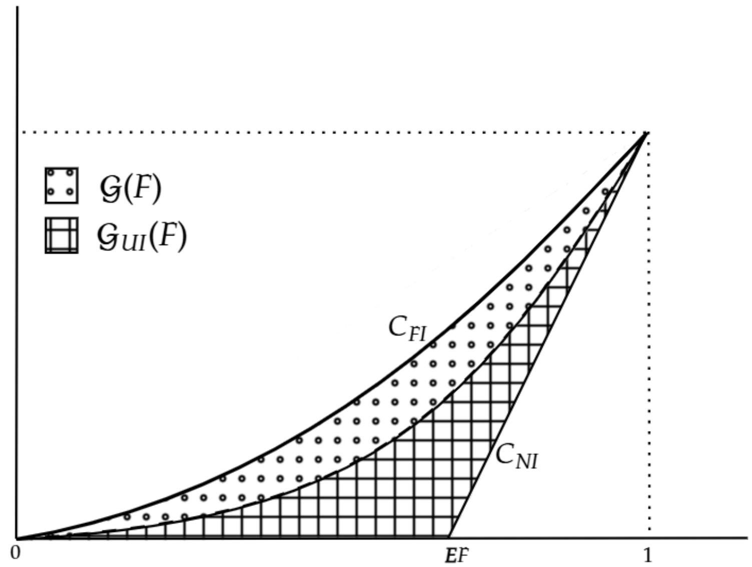

Figure 1: Feasible Demands

The above are information functionals for prior F, Dirac mass on $\mathbf{E}F$, and a fully informed buyer (Dirac mass on 0 and 1). The convex functions in the region between C_{NI} and C_{FI} constitute all possible information functionals generated by some demand with expected valuation $\mathbf{E}F$, as shown by Gentzkow and Kamenica (2016). The functional C_F partitions this region into demands that are mean preserving spread of F, given by $\mathcal{G}(F)$, and demands that are mean preserving contraction of F, given by $\mathcal{G}_{\text{UI}}(F)$.

Equilibrium Outcomes

We will assume that the seller can dictate the terms of trade between the buyer and seller. Thus, we require that buyer trades if and only if his posterior willingness to pay weakly exceeds p.[1]

The seller is a monopolist and her profit-maximizing price against a market demand G is

$$p \in \text{argmax}_{s \in [0,1]} \, s(\, 1 - G(s) \, + \Delta(G, s))$$

[1]For the usual reasons we will break ties by assuming that the buyer purchases the good when indifferent.

Where $\Delta(G, s)$ represents the size of the atom at s for the distribution G.[2]

$$\Delta(G, s) := \lim_{x \downarrow s} G(x) - \lim_{x \uparrow s} G(x)$$

An *equilibrium profile* comprises of the tuple (G, p) such that 1) $G \in \mathcal{G}(F)$, 2) $p \in \operatorname{argmax}_{s \in [0,1]} s(\, 1 - G(s) \, + \Delta(G, s))$.
Each equilibrium profile has its corresponding *outcome* (κ, π) where κ is the consumer surplus and π is the producer surplus given by

$$\kappa := \int_p^1 (s - p) dG(s)$$

$$\pi := p(1 - G(p) + \Delta(G, p))$$

In particular, $\pi \leq p$ and $\kappa \leq \mathbf{E}F - \pi$.

Efficient Demands

An equilibrium profile is *efficient* if it generates the maximum total surplus given by $\kappa + \pi = \mathbf{E}F$. As trade always generates a positive surplus, efficiency requires inducing the maximum probability of trade subject to the constraint that the buyer has no less information than he had initially. The following lemma characterizes efficient demands that can be generated by information design.

Lemma 1. *An equilibrium profile (G, p) is efficient if and only if $supp(G) \cap (0, p) = \varnothing$.*

Proof. Fix some equilibrium outcome (G, p). If $\text{supp}(G) \cap (0, p) = \varnothing$ then conditional on $\theta = 1$ the trade happens with probability one. The sufficiency part of the claim follows from the law of total expectation.

For necessity, consider $\text{supp}(G) \cap (0, p) \neq \varnothing$, then we can construct a new cdf. The designer can provide additional information to all buyers with willingness to pay in $(0, p)$. For any type $\mu \in (0, p)$, the additional information induces a posterior valuation 0 with probability $1 - \alpha$, a valuation p with probability $(1 - \delta)\alpha$, and valuation $p + \varepsilon$ with probability $\delta\alpha$. The mean preserving constraint then requires $\alpha = \frac{\mu}{p + \delta\varepsilon}$.

The proof is complete if there exists $\varepsilon, \delta > 0$ such that the seller still finds it optimal to set the price p. We have increased the mass of buyers with valuation p by

[2] We adopt this notation from Roesler and Szentes (2017).

$\alpha(1-\delta)\int_0^p dG(s)$. The mass of buyers willing to buy at a price $p_1 \in (p, p+\varepsilon]$ is increased by $\alpha\delta\int_0^p dG(s)$. The producer surplus from charging a price above $p+\varepsilon$ is unchanged. As G was an outcome, we only need to verify whether the transformed demand creates incentives to price at some $p_1 \in (p, p+\varepsilon]$. Under the transformed demand, the seller's payoff from charging price p is

$$\pi + p\alpha(1-\delta)\int_0^p dG(s)$$

The seller's payoff from charging a price $p_1 \in (p, p+\varepsilon]$ is bounded above by

$$\pi + (p+\varepsilon)\alpha\delta\int_0^p dG(s)$$

The profit maximizing price is p whenever $p\frac{1-2\delta}{\delta} \geq \varepsilon > 0$. □

Note that the construction in the proof of Lemma 1 strictly increases both the consumer and producer surplus, thus an **equilibrium profile is *efficient* if and only if it is on the *Pareto frontier***. Thus, it is without loss to focus on efficient outcomes if one is interested in determining producer or consumer optimal equilibrium outcomes.

In general, many different market demands G can be feasible and lead to the same equilibrium price and outcome. These demands could be incomparable with respect to the convex order. We will focus on the following efficient demands determined by a market price p, a producer surplus π, and an expected valuation m.

$$H_p^\pi(s;m) = \begin{cases} 1-\frac{\pi}{p} & s < p \\ 1-\frac{\pi}{t(p,\pi,m)} & s = p \\ 1-\frac{\pi}{s} & 1 > s \geq t(p,\pi,m) \\ 1 & s = 1 \end{cases}$$

Where $t(p,\pi,m)$ is implicitly defined as the solution to the following whenever it exists [3]

$$\pi + \int_t^1 \frac{\pi}{s}ds + p\pi\left[\frac{1}{p} - \frac{1}{t}\right] = m \tag{1}$$

Whenever $m = \mathbf{E}F$ we will drop m from the notation and just write H_p^π and

[3] If $p < 2 - \frac{m}{\pi}$, then Equation 1 holds for no $t < 1$. In this case if $\pi \leq m < \pi(2-p)$ we let $t(p,\pi) := 1$ and the distribution H_p^π is described by three mass points $H_p^\pi(0) = 1 - \frac{\pi}{p}$, $\Delta(H_p^\pi, 1) = \frac{m-\pi}{1-p}$, and $\Delta(H_p^\pi, p) = 1 - \Delta(H_p^\pi, 1) - H_p^\pi(0)$.

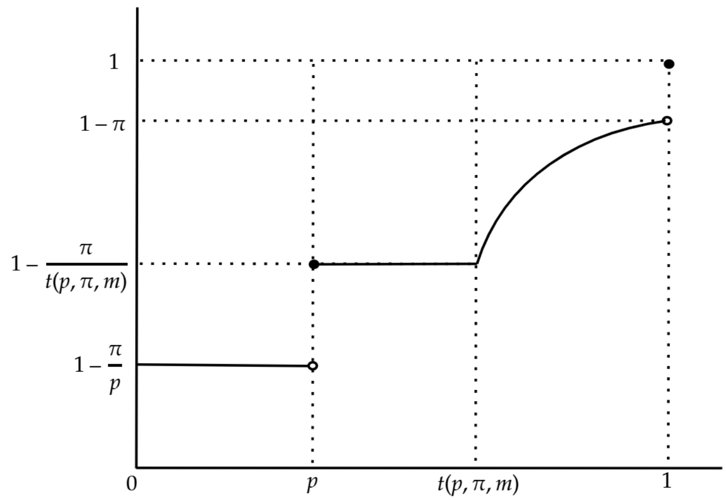

Figure 2: $H_p^\pi(s;m)$ when Equation 1 holds for $t \leq 1$

$t(p,\pi)$. Note for later that the consumer surplus κ from any implementable H_p^π is given by $\kappa = \mathbf{E}F - \pi$.

Define

$$\mathcal{H}(F) := \{G \in \mathcal{G}(F) \mid \exists\, \pi, p \text{ such that } G = H_p^\pi\}$$

These demands are iso-elastic (or unit elastic) on their support. For a given $H \in \mathcal{H}(F)$, the seller is indifferent between charging any positive price on the support of H. We will show that the above set of demands is sufficient to generate all possible efficient outcomes. Moreover, any efficient equilibrium demand can be transformed into a demand from $\mathcal{H}(F)$ through some information policy without affecting the welfare and price.

Lemma 2. *If (G, p) is an efficient equilibrium profile with corresponding outcomes (κ, π) then (H_p^π, p) is an outcome equivalent equilibrium profile. Moreover $H_p^\pi \succeq_{mps} G$.*

Proof. First we show the claim for a profile (G, p) with producer surplus π such that $p \leq 2 - \frac{\mathbf{E}F}{\pi}$. Let $\widetilde{G}$ be a CDF such that $\Delta(\widetilde{G}, 1) = \frac{\mathbf{E}F - p\int_p^1 dG(s)}{1-p}$, $\Delta(\widetilde{G}, p) = \frac{\int_p^1 dG(s) - \mathbf{E}F}{1-p}$, and $\widetilde{G}(0) = G(0)$. The CDF $\widetilde{G}$ is constructed by type-by-type garbling all types μ in the interval $[p, 1]$ into posteriors supported on $\{p, 1\}$, thus $\widetilde{G} \succeq_{mps} G$. As $p \leq 2 - \frac{\mathbf{E}F}{\pi}$ it must be that $\Delta(\widetilde{G}, 1) \leq \pi$. and $p = \operatorname{argmax}_{[0,1]} s(1 - \widetilde{G}(s) +$

$\Delta(\widetilde{G}, s))$. In particular, $(\widetilde{G}, p)$ is an equilibrium profile with producer surplus π. Note that as (G, p) is an equilibrium profile it must be that $\pi \leq \mathbf{E}F$, thus $\widetilde{G} = H^{\pi}_{p}$.

Let $p > 2 - \frac{\mathbf{E}F}{\pi}$. As (G, p) is an equilibrium profile, we get that $p \in \operatorname{argmax}_s s(1 - G(s) + \Delta(G, s))$. This implies that $G(s) \geq 1 - \frac{\pi}{s}$ for all $s \in [p, 1]$ and $\Delta(G, 1) \leq \pi$. Thus the curve G lies weakly above the curve $H^{\pi}_{p}(\,\cdot\,; \pi(1 - \ln p))$. Where $\pi(1 - \ln p)$ is the maximum possible expected valuation of a demand for which the profit maximizing price is p and which delivers a producer surplus of π. As (G, p) is an outcome we get that $\mathbf{E}G = \mathbf{E}F \leq \pi(1 - \ln p)$. By assumption $(2 - p)\pi < \mathbf{E}F$. Thus there exists a unique $t(p, \pi) \in [p, 1]$ which satisfies Equation 1 and H^{π}_{p} is well defined.

As both G and H^{π}_{p} are efficient demands for producer surplus π and price p we get that $G(0) = H^{\pi}_{p}(0) = 1 - \frac{\pi}{p}$. Since $G(s) \geq 1 - \frac{\pi}{s}$ for all $s \in [p, 1]$, we get $\frac{\pi}{p} - \frac{\pi}{t(p,\pi)} \geq \Delta(G, p)$ as otherwise G lies strictly above H^{π}_{p} on $(p, 1)$ which can not happen as G and H^{π}_{p} have the same mean. Moreover, as G lies weakly above H^{π}_{p} on the interval $[t(p, \pi), 1]$, Equation 1 (mean is preserves) along with $G(0) = H^{\pi}_{p}(0)$ further imply that there exists $p_1 \in (p, t(p, \pi))$ such that $G(s) < H^{\pi}_{p}(s)$ for $s \in [p, p_1)$ and $G(s) \geq H^{\pi}_{p}(s)$ for $s \in [p_1, t(p, \pi))$. In particular, we have shown G crosses H^{π}_{p} once and from below.

The above, along with Equation 1 implies that $H^{\pi}_{p} \succeq_{mps} G$. As (G, p) is an equilibrium profile we get that $H^{\pi}_{p} \succeq_{mps} F$. By construction $p \in \operatorname{argmax}_{s \in [0,1]} s(1 - H^{\pi}_{p}(s) + \Delta(H^{\pi}_{p}, s))$, thus (H^{π}_{p}, p) is an equilibrium profile. It induces the same producer surplus π, price p, probability of trade $1 - \Delta(G, 0)$ and consumer surplus $\mathbf{E}F - \pi$. □

When the buyer and seller are initially uninformed as in Roesler and Szentes (2017), an efficient outcome involves trade with probability one at a price below the prior expected valuation. In particular, given an efficient outcome with price p, the producer surplus is $\pi = p$ and the consumer surplus is $\mathbf{E}F - p$. Thus, the outcome is determined by the equilibrium price.[4] In contrast to this, when the buyer is initially privately informed, an efficient outcome is not completely determined by the equilibrium price and the probability of trade might be bounded

[4]Similarly, when the buyer and seller are symmetrically informed, the efficient outcome conditional on buyer's prior expected valuation μ entails trade with probability one at a price $p < \mu$. The total surplus conditional on buyer's prior expected valuation μ is μ and the consumer surplus is $\mu - p$.

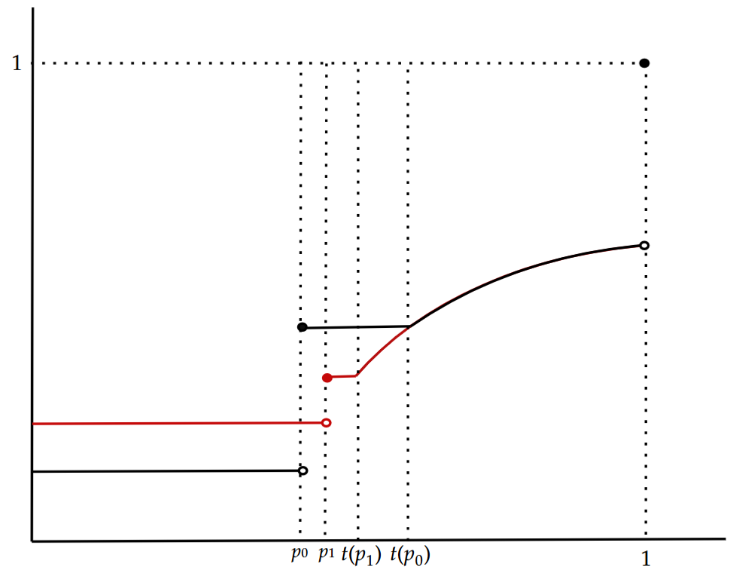

Figure 3: Single crossing between $H^{\pi}_{p_0}$ in black and $H^{\pi}_{p_1}$ in red.

away from 1. Moreover, multiple prices can support the same efficient outcome. The decoupling of price and welfare is only possible because the buyer and the seller are initially asymmetrically informed.

Fixing an outcome, how does the change in the amount of information affect the equilibrium prices? The result below highlights comparative statics between the amount of information provided to the buyer and the price level in equilibrium, keeping the expected consumer and producer surplus fixed at some efficient level. This might be relevant, say, for a policymaker who is concerned about some exogenous spill-over effect of high prices on the general economy, as well as concerned about unnecessarily intervening.

Theorem 1. *For any $p_0 < p_1$ and π, if $(H^{\pi}_{p_0}, p_0)$ and $(H^{\pi}_{p_1}, p_1)$ are equilibrium profiles then $H^{\pi}_{p_1} \succeq_{mps} H^{\pi}_{p_0}$.*

Proof. By definition to prove $H^{\pi}_{p_1} \succeq_{mps} H^{\pi}_{p_0}$ it suffices to show that $t(p_0, \pi) \geq t(p_1, \pi)$. By Equation 1 we get that

$$p_0 \left(\frac{\pi}{p_0} - \frac{\pi}{t(p_0, \pi)} \right) + \int_{t(p_0,\pi)}^{1} \frac{\pi}{s} \, ds = p_1 \left(\frac{\pi}{p_1} - \frac{\pi}{t(p_1, \pi)} \right) + \int_{t(p_1,\pi)}^{1} \frac{\pi}{s} \, ds$$

$$\implies p_0\left(\frac{\pi}{p_0}-\frac{\pi}{t(p_0,\pi)}-\frac{\pi}{p_1}+\frac{\pi}{t(p_1,\pi)}\right)=(p_1-p_0)\left(\frac{\pi}{p_1}-\frac{\pi}{t(p_1,\pi)}\right)+\int_{t(p_1,\pi)}^{t(p_0,\pi)}\frac{\pi}{s}\,ds$$

$$\implies p_0\left(\frac{\pi}{p_0}-\frac{\pi}{p_1}\right)+p_0\int_{t(p_1,\pi)}^{t(p_0,\pi)}\frac{\pi}{s^2}ds=(p_1-p_0)\left(\frac{\pi}{p_1}-\frac{\pi}{t(p_1,\pi)}\right)+\int_{t(p_1,\pi)}^{t(p_0,\pi)}\frac{\pi}{s}\,ds$$

$$\implies p_0\left(\frac{\pi}{p_0}-\frac{\pi}{p_1}\right)=(p_1-p_0)\left(\frac{\pi}{p_1}-\frac{\pi}{t(p_1,\pi)}\right)+\int_{t(p_1,\pi)}^{t(p_0,\pi)}\left(\frac{\pi}{s}-p_0\frac{\pi}{s^2}\right)\,ds$$

$$\implies \frac{p_1-p_0}{t(p_1,\pi)}=\int_{t(p_1,\pi)}^{t(p_0,\pi)}\left(\frac{1}{s}-p_0\frac{1}{s^2}\right)\,ds$$

As $t(p_1,\pi), t(p_0,\pi)\geq p_0\geq 0$ we get that the above equation holds only if $t(p_0,\pi)\geq t(p_1,\pi)$. (See Figure 3) □

More informed buyers are less (more) likely to trade, conditional on the item having value $\theta=0$ ($\theta=1$). Thus, keeping the total surplus and the share of surplus fixed, the seller sets a higher price p for more informed buyers.

Buyer Optimal

In this section, we determine the buyer optimal outcome. From Lemma 1, we know that the buyer optimal outcome must be efficient. The following lemma presents a sufficient condition under which the consumer surplus can be strictly improved.

Proposition 1. *Given some efficient equilibrium profile (G,p) with outcome (κ,π), if $p<2-\frac{\mathbf{E}F}{\pi}$ then there exist $p_0<p$ and $\pi_0<\pi$ such that $H_{p_0}^{\pi_0}\succeq_{mps} G$ and $(H_{p_0}^{\pi_0},p_0)$ is an equilibrium profile.*

Proof. By the argument in Lemma 2, we can restrict attention to the distribution H_p^π. Define the following functions $\Pi(s):=s(1-H_p^\pi(0))$, and $W(s):=\Delta(H_p^\pi,1)+\frac{p-s}{1-s}\Delta(H_p^\pi,p)$. As $p<2-\frac{\mathbf{E}F}{\pi}$ and (H_p^π,p) is an equilibrium profile we get that $W(p)<\Pi(p)$ and $W(0)>\Pi(0)$. Thus, by continuity and the intermediate value theorem, there is a price $0<p_0<p$ such that $\Pi(p_0)=W(p_0)$.

Now define a CDF $\widetilde{G}$ such that $\widetilde{G}(0)=H_p^\pi(0)$, $\Delta(\widetilde{G},p_0):=\frac{1-p}{1-p_0}\Delta(H_p^\pi,p)$, and $\Delta(\widetilde{G},1):=\Delta(H_p^\pi,1)+\frac{p-p_0}{1-p_0}\Delta(H_p^\pi,p)$. By construction $\widetilde{G}\succeq_{mps} H_p^\pi$ and $(\widetilde{G},p_0)$ is an equilibrium profile. The conclusion follows from noting that $H_{p_0}^{\pi_0}=\widetilde{G}$, with $\pi_0:=p_0(1-\widetilde{G}(0)))<p(1-H_p^\pi(0))$. □

The above lemma identifies a relationship between the market price p and the equilibrium producer surplus π under which the designer can construct information policies that lead to strictly better consumer surplus.

Lemma 3. *For any efficient equilibrium profile (G, p) with outcome $(\kappa,\ \pi)$ there exists a unique $p_\pi \geq p$ such that $t(p_\pi, \pi) = p_\pi$, $H^\pi_{p_\pi} \succeq_{mps} G$, and $(H^\pi_{p_\pi}, p_\pi)$ is an outcome equivalent equilibrium profile.*
Moreover, for any equilibrium profile (G, p) the pair $(H^\pi_{p_\pi}, p_\pi)$ is also an equilibrium profile with the same producer surplus and $H^\pi_{p_\pi} \succeq_{mps} G$.

Proof. By Lemma 2 we get that (H^π_p, p) is an equilibrium profile such that $H^\pi_p \succeq_{mps} G$. If $p < t(p, \pi)$, then consider some $\varepsilon > 0$ such that $p + \varepsilon = t(p + \varepsilon, \pi)$. By Theorem 1 we get that $t(p, \pi) > t(s, \pi)$ for all $s > p$ whenever it is well defined, thus an $\varepsilon > 0$ that satisfies the required equality exists. The uniqueness follows from the fact that Equation 1 has a unique solution with $t(p, \pi) = p$. In particular, this solution is $p_\pi = \exp\left(1 - \frac{\mathbf{E}F}{\pi}\right)$. Finally, note that any price $s > p_\pi$ can not be supported by an efficient equilibrium profile with producer surplus π as Equation 1 can not be satisfied.

To prove the second statement, consider an equilibrium profile (G, p), not necessarily efficient, with outcome (κ, π). As π is maximum producer surplus under demand G we get that $1 - G(s) + \Delta(G, s) \leq \pi/s$ for all $s \in [0.1]$. In particular $G(s) \geq 1 - \frac{\pi}{s}$ for all $s \in [0.1]$. As (κ, π) is an equilibrium outcome, it must be that $\pi \leq \mathbf{E}F$. This implies that $p_\pi = \exp\left(1 - \frac{\mathbf{E}F}{\pi}\right) \in (0, 1]$. By construction $\mathbf{E}H^\pi_{p_\pi} = \mathbf{E}F = \mathbf{E}G$, combining this with the definition of $H^\pi_{p_\pi}$ we get that G crosses $H^\pi_{p_\pi}$ once and from below (See Figure 4). Thus $H^\pi_{p_\pi} \succeq_{mps} G \succeq_{mps} F$ completing the proof. □

Let $\pi^* := \inf\{\pi \mid \exists\ p \text{ such that } (H^\pi_p, p) \text{ is an equilibrium profile }\}$, and let

$$p^* := p_{\pi^*}.$$

In particular by Lemma 3 and Equation 1 we get that $p^* = \exp\left(1 - \frac{\mathbf{E}F}{\pi^*}\right)$. The producer surplus corresponding to the buyer's optimal outcome is given by π^* and is a solution to the following

$$\inf_{\pi \in [0,1]} \pi \quad \text{subject to} \quad \int_0^x \left(H^\pi_{p_\pi}(s) - F(s)\right)\, ds \geq 0 \quad \text{for all } x \in [0, 1]$$

Where

$$p_\pi = \exp\left(1 - \frac{\mathbf{E}F}{\pi}\right)$$

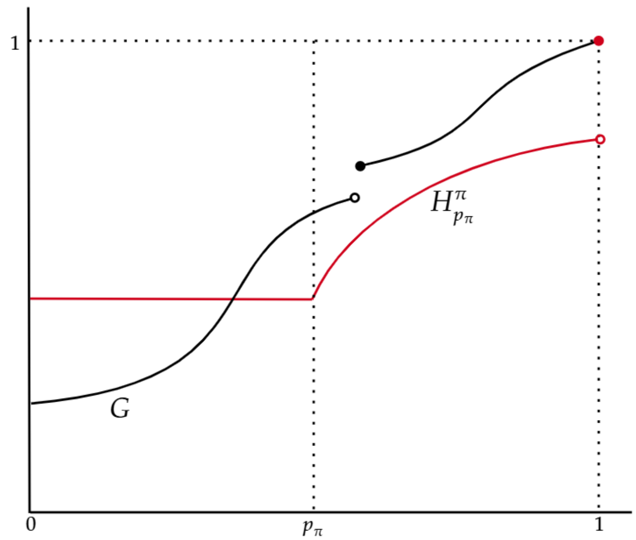

Figure 4: Single crossing between $H^\pi_{p_\pi}$ in red and some generic equilibrium demand G in black.

Theorem 2. *If an equilibrium profile (G, p) is consumer optimal then it is outcome equivalent to $(H^{\pi^*}_{p^*}, p^*)$ and $H^{\pi^*}_{p^*} \succeq_{mps} G$.*

Proof. By Lemma 3 we get that there exists some $p_1 \geq p$ such that $(H^\pi_{p_1}, p)$ is an equilibrium profile and $H^\pi_{p_1} \succeq_{mps} G$. As G is consumer optimal, it is also efficient by Lemma 1. Thus, it must be that (G, p) generates producer surplus π^*. Finally by Theorem 1 and Lemma 3, it must be that $p^* = t(p^*, \pi^*)$ and $H^{\pi^*}_{p^*} \succeq_{mps} G$. □

Example : Let $F \sim \text{Unif}[0,1]$. By Theorem 2 we get that the optimal consumer surplus is given by

$$\inf_{\pi \in [0,1]} \pi \quad \text{subject to} \quad \int_0^x \left(H^\pi_{p_\pi}(s) - s \right) \, ds \geq 0 \quad \text{for all } x \in [0,1]$$

After solving the above problem numerically, we get the consumer surplus maximizing level of $\pi^* \simeq 0.204$.

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