Learning about Risk

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Abstract

We study efficient and sender surplus-maximizing information design in selection markets, specializing our exposition to credit markets. A borrower (sender) designs information about the riskiness of his project and seeks to finance his project through a monopolist lender. We characterize the borrower-optimal learning when the lender's utility depends on both the demand of borrowers and on the risk composition of the borrowers. We use this characterization to describe relation between the set of efficient equilibrium welfare, risk dispersion and the shape of repayment contract (securities).

Introduction

We study information design in selection markets, where a sender interacts with a receiver whose payoff from trade depends jointly on the terms of trade and on the sender's underlying type. We specialize this framework to credit markets, where repayment terms affect both the demand for financing and the risk composition of borrowers who accept.

A borrower (sender) chooses how much to learn about the riskiness of his project by committing to an experiment and privately observes its realization. The lender (receiver) observes the experiment but not its realization and offers a concave repayment contract in response to the induced distribution of posterior beliefs. The borrower then accepts financing or exercises his outside option. If financed, the

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project return is realized, and repayment is subject to the borrower's limited liability.

We characterize efficient and sender-optimal information structures by identifying, for every equilibrium receiver surplus, the least informative experiment that implements it. Our theoretical contribution is a selection-market analogue of buyer-optimal learning in Roesler and Szentes (2017). For a fixed receiver surplus, we represent the distribution of posterior beliefs by its *call function* and construct the least-informative implementation from the pointwise lower envelope of implementing call functions and its convex minorant.

We use this characterization to study how selection shapes implementable welfare. When the receiver's payoff falls more rapidly as the composition of trading senders shifts toward less profitable types, the set of implementable efficient payoffs expands toward the sender. In the credit-market application, we show that greater dispersion in project risk and greater concavity of repayment both expand the set of implementable borrower–lender surplus divisions through this channel.

Literature Review

Roesler and Szentes (2017) study buyer learning when the seller's payoff from trade depends only on whether trade occurs. Kartik and Zhong (2026) allow the seller's payoff to vary with buyer type, but the value of trading with a given type is independent of the terms offered. We instead study environments in which the contract itself changes the value of trading with a given type, so contract choice determines both participation and the strength of selection.

Credit markets provide a natural application. As in Stiglitz and Weiss (1981), repayment terms affect borrower selection and lender returns. We study endogenous borrower learning and characterize how project-risk dispersion and repayment structure change the resulting selection problem.

Model

We study a financing relationship between a borrower (he) and a monopolistic lender (she). Financing a project requires an investment of one unit. The borrower can forgo financing the project in favor of an outside option $1 > R_0 > 0$.

Project Returns

The project has a binary state θ in $\{0,1\}$. Conditional on state θ in $\{0,1\}$, the project return R is distributed according to G_θ. The two states have the same expected return,

$$\mathbb{E}_{G_\theta}[R] = \overline{R}.$$

State 1 is riskier than state 0 in convex order,

$$G_0 \preceq_{\text{cx}} G_1. \tag{1}$$

For two distributions H and G, the relation $H \preceq_{\text{cx}} G$ holds if and only if for every real valued convex function h we have the following

$$\int h(s)\, dG(s) \geq \int h(s)\, dH(s)$$

Thus the two states differ only in risk and have the same expected return. The common prior probability of the risky state is m in $(0,1)$. In particular, a higher expectation $\mathbb{E}\theta$ corresponds to a riskier borrower.

We assume that the expected return of the project net of the investment cost is greater than the borrower's outside option.

$$\Delta := \overline{R} - 1 - R_0 > 0. \tag{2}$$

Because the expected project return is the same in both states, Δ is the total expected gain from financing net of the borrowers outside option at any belief about θ . In particular, financing is efficient at any beleif μ in $\Delta(\{0,1\})$.

Repayment Contracts

The lender chooses a *contract* r from an interval [0,1]. Under contract r, the lender receives a return-contingent repayment

$$T(R,r).$$

The borrower receives the residual return

$$R - T(R,r).$$

We assume the contract family is *rich* such that for each type μ there exists a contract r under which μ is indifferent between the contract and his outside option.

We will focus on repayment contracts that are concave in R for every r. Concavity of repayments implies that if a borrower accepts a given contract $T(\cdot, r)$ over his outside option, then so does a riskier borrower. Additionally, we require that T increases in both its arguments, representing higher repayments for higher returns and for higher contract terms.

In particular, we impose the following conditions on the contract family.

Assumption 1. *For every r in $[0,1]$, the repayment function $T(\cdot, r)$ is non-decreasing and concave in R, and*

$$0 \leq T(R, r) \leq R.$$

Moreover, $T(R, r)$ is continuous and non-decreasing in r for every R, and $\mathbb{E}_{G_\theta}[T(R, r)]$ is strictly increasing.

For example, a contract with a fixed collection amount and limited liability corresponds to $T(R, r) = \min\{R, k(r)\}$ for some increasing positive function k, satisfies the Assumption 1.

The first inequality in the assumption represents a non-negative ex-post payment from the borrower. The second inequality is economically more relevant and represents the **limited liability** of the borrower; the ex-post payment from the borrower to the lender is at most the gross realized return of the project.

For θ in $\{0,1\}$ define the borrower's expected residual payoff under contract r by

$$u_\theta(r) := \mathbb{E}_{G_\theta}\left[R - T(R, r)\right].$$

The lender's expected net payoff in state θ is

$$\ell_\theta(r) := \mathbb{E}_{G_\theta}\left[T(R, r)\right] - 1 = \overline{R} - u_\theta(r) - 1.$$

Continuity of $T(R, r)$ in r implies that $u_\theta(r)$ is continuous in r for each state.

Because $T(\cdot, r)$ is concave, the borrower's residual claim $R - T(R, r)$ is convex. The Equation 1 therefore implies

$$u_1(r) \geq u_0(r) \qquad \text{for every } r \in [0,1]. \tag{3}$$

A borrower with posterior belief $\mu \in [0,1]$ assigns probability μ to state 1. His expected residual payoff from contract r is

$$u_\mu(r) := (1-\mu)u_0(r) + \mu u_1(r).$$

The borrower accepts financing whenever $u_\mu(r) \geq R_0$. With ties broken in favor of acceptance.

By Assumption 1, the expected repayment is strictly increasing in r under either state, so $u_\theta(r)$, and hence $u_\mu(r)$, is strictly decreasing in r. Continuity and strict monotonicity of $u_\mu(r)$ then imply that for each μ there exists a unique contract $\hat{r}(\mu)$ at which type μ borrower is indifferent between financing and his outside option.[1] In particular, the *reservation contract* $\hat{r}(\cdot)$ satisfies

$$(1-\mu)u_0(\hat{r}(\mu)) + \mu u_1(\hat{r}(\mu)) = R_0. \tag{4}$$

For $s > \mu$, Equation 3 implies $u_s(r) \geq u_\mu(r)$ for every r. Every contract accepted by type μ is also accepted by type s, and therefore $\hat{r}(s) \geq \hat{r}(\mu)$. Thus, under assumption 1, $\hat{r}$ is continuous and weakly increasing.

If type μ is the marginal borrower then the slope of the borrower's utility is 0 for $s < \mu$ and for $s \geq \mu$ it is given by

$$D(\mu) := u_1(\hat{r}(\mu)) - u_0(\hat{r}(\mu)) \geq 0.$$

If type μ is the marginal borrower then the borrower's indirect utility is constant at the outside option for $s < \mu$, while for $s \geq \mu$ its slope with respect to s is $D(\mu)$. The function D describes how riskier borrowers accumulate rents by being pooled with less risky borrowers.

Moreover, either $\hat{r}$ is constant on $[0,1]$ or it is strictly increasing. To see this, suppose $D(\mu) = 0$ for some μ. By Equation 4,

$$u_\theta(\hat{r}(\mu)) = R_0.$$

The same contract then leaves every posterior type indifferent, so $\hat{r}$ is constant on $[0,1]$. Otherwise $D(\mu) > 0$ for every μ. For $s > \mu$,

$$u_s(\hat{r}(\mu)) = R_0 + (s-\mu)D(\mu) > R_0,$$

the borrower's utility is decreasing in r, thus $\hat{r}(s) > \hat{r}(\mu)$.

If $\hat{r}$ is constant, the lender faces no selection margin and private learning does not affect which borrowers accept a contract. The information-design problem below

[1] Existence follows, without loss of generality, from re-parameterizing the contract space such that the contract terms satisfy $\hat{r}(0) = 0$ and $\hat{r}(1) = 1$.

is therefore immediate in that case. We proceed with the non-constant case when D is strictly positive.

We will let $\hat{\mu}(\cdot)$ to represent the inverse of $\hat{r}(\cdot)$, where $\hat{\mu}(r)$ is the marginal borrower type that accepts the contract r.

The Lender's Problem

At the reservation contract of type s, Equation 4 and Equation 1 imply

$$\ell_{\mu}(\hat{r}(\mu)) = \overline{R} - 1 - R_0 = \Delta.$$

The lender surplus from financing a type $s > \mu$ with the contract $\hat{r}(\mu)$ is

$$\begin{aligned} \ell_s(\hat{r}(\mu)) &= \Delta - (s-\mu)\left[u_1(r(\mu)) - u_0(r(\mu))\right] \\ &= \Delta - (s-\mu)D(\mu). \end{aligned}$$

Thus a higher posterior type is more willing to accept the contract but, conditional on acceptance, is less valuable to the lender.

Learning

Before the lender chooses a contract, the borrower commits to an experiment ρ about the state. The realization of the experiment is observed privately by the borrower. The lender observes the experiment, but not its realization. The lender then offers a contract with terms r and the borrower accepts the contract terms or exercises his outside option.

An experiment induces a distribution F over posterior beliefs μ. We refer to this distribution of posterior expectation as a *credit demand*. Because θ is binary, an arbitrary distribution on $[0, 1]$ is induced by some experiment if and only if it is Bayes plausible,

$$\int_0^1 \mu \, dF(\mu) = m.$$

Let $\mathcal{F}(m)$ represent the set of all *feasible credit demand*.

The reservation contracts are strictly increasing and the contract $\hat{r}(\mu)$ is accepted exactly by types in $[\mu, 1]$. It is therefore without loss to index the lender's choice by the cutoff μ. For a feasible demand F, the lender's expected payoff from a contract $\hat{r}(\mu)$ is given by

$$\Pi_F(\mu) := \int_\mu^1 \ell_s(\hat{r}(\mu))\, dF(s) = \int_\mu^1 \Delta - (s-\mu)D(\mu)\, dF(s) \tag{5}$$

The payoff of the borrower depends on his posterior belief, and the lender's payoff (and her choice) depends on the distribution of the borrower's posterior belief (credit demand) F. In particular, it is without loss to map each experiment to the credit demand it induces.

Selection Market

A *selection market* is a market in which the service provider's cost depends both on the quality of service and the composition of buyers. The credit market is an example of a selection market, the lender's cost of service depends on both the contract terms and the composition of financed borrowers.

For a feasible credit demand F, define its *call function*

$$\phi_F(\mu) := \int_\mu^1 (s-\mu)\, dF(s) = \mathbb{E}_F\left[(s-\mu)^+\right].$$

Its left derivative satisfies

$$-\phi'_{F,-}(\mu) = F([\mu,1]).$$

By Equation 5 the lender's payoff from offering contract $\hat{r}(\mu)$ against demand F,

$$\Pi_F(\mu) = \underbrace{\Delta\left[-\phi'_{F,-}(\mu)\right]}_{\text{Gross Contribution}} + \underbrace{-D(\mu)\phi_F(\mu)}_{\text{Rents from Selection}}. \tag{6}$$

The decomposition in Equation 6 highlights central features of the credit market. The first term is the value of serving the mass of accepting borrowers times the value from the marginal borrower. The second term is the loss from selection toward posterior types that place greater probability on the risky state. The lender's payoff depends not only on the mass of borrowers that accept financing but also on the composition of the risk profiles of the borrower conditional on accepting the contract. In particular, given a demand F, the lender's payoff from a contract

r depends on both the mass and the conditional expectation the right-tail of the demand above the marginal borrower $\hat{\mu}(r)$.

A call function generated by a feasible demand is convex, non-increasing, and satisfies

$$\phi(0) = m, \qquad \phi(1) = 0,$$

with slopes in $[-1, 0]$. Let the set of such function be given by $\Phi(m)$. Importantly, the sets $\Phi(m)$ and $\mathcal{F}(m)$ are isomorphic, see for example Gentzkow and Kamenica (2016).

Although we focus on adverse selection in a pure risk credit market. The decomposition Equation 5 and the use of *call functions* to study information design apply to adverse selection markets more generally, including markets where total surplus from participation depends on the sender's type. Moreover, analogous results can be derived in markets with advantageous selection by using *put functions* $\psi_F(\mu) = \int_0^\mu (\mu - s) dF(s)$.

Equilibrium

An equilibrium profile is a pair (F, s) such that F is Bayes-plausible and

$$s \in \operatorname{argmax}_{s \in [0,1]} \Pi_F(s).$$

The corresponding lender surplus is

$$\pi = \Pi_F(s).$$

The borrower's surplus net of his reservation, R_0, is

$$\kappa := \int_s^1 \left[u_\mu(\hat{r}(s)) - R_0 \right] dF(\mu).$$

Main Result

We take the lender surplus π as fixed. Our first result identifies the least informative posterior distribution that implements a given equilibrium seller surplus. We will later use this to study the buyer-optimal outcome and comparative statics.

For an implementable lender surplus π, let

$$\Phi_\pi := \left\{ \phi \in \Phi(m) : \max_{s \in [0,1]} \left\{ \Delta[-\phi'_-(s)] - D(s)\phi(s) \right\} = \pi \right\}.$$

Define the pointwise lower envelope of Φ_π implementations by

$$\phi_\pi^{\inf}(s) := \inf_{\phi \in \Phi_\pi} \phi(s),$$

and let $\widehat{\phi}_\pi$ denote the greatest convex function lying below $\phi_\pi^{\inf}$, in other words $\widehat{\phi}_\pi$ is the convex minorant of $\phi_\pi^{\inf}$. Below we show that $\widehat{\phi}_\pi$ is in Φ_π, and thus the corresponding demand $\widehat{F}_\pi$ is the least informative demand implementing π.

Theorem 1. *Suppose Assumption 1 holds. If π is an equilibrium lender surplus,then the convex minorant $\widehat{\phi}_\pi$ of $\phi_\pi^{\inf}$ is a feasible call function and implements lender surplus π. Moreover, if $\widehat{F}_\pi$ denotes the posterior distribution generated by $\widehat{\phi}_\pi$, then $\widehat{F}_\pi \preceq_{\mathrm{cx}} F$ for every posterior distribution F that implements π . Thus, $\widehat{F}_\pi$ is the least informative implementation of π.*

Proof. See Appendix A. □

Theorem 1 shows that the convex minorant $\widehat{\phi}_\pi$ is a feasible call function and implements the lender surplus π. Let

$$[\underline{\mu}_\pi, \overline{\mu}_\pi] := \mathrm{co}\Big(\mathrm{supp}\, \widehat{F}_\pi\Big)$$

be the smallest closed interval containing the support of the demand generated by $\widehat{\phi}_\pi$. Since its mean is m,

$$0 \le \underline{\mu}_\pi \le m \le \overline{\mu}_\pi \le 1.$$

For $\mu < \underline{\mu}_\pi$ and $\mu > \overline{\mu}_\pi$ the demand $\widehat{F}_\pi$ has no mass.

On the interval $[\underline{\mu}_\pi, \overline{\mu}_\pi]$, the pointwise minimality of $\widehat{\phi}_\pi$ among ϕ in Φ_π implies that the lender's profit is constant and equal to π. To see this consider for contradiction that the lender's payoff constraint is slack in the interior $(\underline{\mu}_\pi, \overline{\mu}_\pi)$. Suppose the lender constraint were strict at some $s_0 \in (\underline{\mu}_\pi, \overline{\mu}_\pi)$. Because s_0 lies in the interior of the convex hull of the support, there is posterior mass on both sides of s_0 (although s_0 itself need not be a support point). Now construct a new perturbed demand by collapsing the mass on a neighborhood of s_0 to its conditional expectation (barycenter). In particular, the right and left neighborhoods whose mass is collapsed is chosen such that the conditional expectation equals s_0. Given the perturbed demand, the lender's payoff from cutoffs above s_0 is weakly lower than under $\widehat{\phi}_\pi$ and hence less than π. Whereas, for the perturbed demand, the lender's payoff from a cutoff below the interval on which the mass is collapsed is unchanged and is thus (weakly) below π. For cutoff below s_0 and in the interval from which the mass is collapsed, the lender surplus is weakly lower than the lender's payoff at cutoff s_0. The lender surplus increases at s_0 under the perturbed demand. We can choose the mass that is collapsed onto s_0 to be arbitrarily small, and thus the increase in the lender's payoff at s_0 is can be made arbitrarily small. By assumption, the lender's payoff at s_0 was strictly less than π. Thus the perturbed call function corresponds to a lender surplus that is at most π. Moreover, by construction, the perturbed call function lies below $\widehat{\phi}_\pi$ which contradicts Theorem 1 (see Appendix A).

Two immediate consequences of the constant lender surplus on $[\underline{\mu}_\pi, \overline{\mu}_\pi)$ are that on this interval, $\widehat{F}_\pi$ is continuous and strictly increasing. An atom in $[\underline{\mu}_\pi, \overline{\mu}_\pi)$ would make the gross contribution in Equation 6 jump while the selection contribution remains continuous. Moreover, a gap in the support would leave the accepting set unchanged as the contract term r increases. Either contradicts constant lender profit under Assumption 1 and right continuity of a feasible demand.

At the upper endpoint, suppose lender profit were strictly below π. On a left neighborhood of $\overline{\mu}$, the lender's profit is π, and as the cutoff approaches $\overline{\mu}_\pi$ from the left it converges to the endpoint payoff, a contradiction. Therefore

$$\widehat{F}_\pi(\{\overline{\mu}_\pi\}) = \frac{\pi}{\Delta} \quad \text{and} \quad (\widehat{\phi}_\pi)'_+(\underline{\mu}_\pi) = -1$$

It follows that $\widehat{\phi}_\pi$ satisfies the following ordinary differential equation for all μ in $[\underline{\mu}_\pi, \overline{\mu}_\pi]$

$$\Delta[-\widehat{\phi}'_\pi(\mu)] - D(\mu)\widehat{\phi}_\pi(\mu) = \pi \tag{7}$$

Using the integration factor $I(\mu) := \exp\left(-\int_\mu^1 \frac{D(s)}{\Delta}\, ds\right)$ and the boundary conditions, we derive the solution to Equation 7 given by

$$\widehat{\phi}_\pi(\mu) = \frac{\pi}{\Delta} \int_\mu^{\overline{\mu}_\pi} \frac{I(s)}{I(\mu)}\, ds, \qquad \underline{\mu}_\pi < \mu < \overline{\mu}_\pi.$$

Putting together, Corollary 1 describes the shape of $\widehat{\phi}_\pi$

Corollary 1. *Suppose Assumption 1 holds. Let $\widehat{\phi}_\pi$ correspond to an equilibrium lender surplus π. Then there exist*

$$0 \le \underline{\mu}_\pi \le m \le \overline{\mu}_\pi \le 1$$

such that

$$\widehat{\phi}_\pi(\mu) = \begin{cases} m-\mu, & \mu \le \underline{\mu}_\pi, \\ \dfrac{\pi}{\Delta}\displaystyle\int_\mu^{\overline{\mu}_\pi} \frac{I(s)}{I(\mu)}\,ds, & \underline{\mu}_\pi < \mu < \overline{\mu}_\pi, \\ 0, & \mu \ge \overline{\mu}_\pi. \end{cases}$$

The interval $[\underline{\mu}_\pi, \overline{\mu}_\pi]$ is the iso-profit region for the minorant $\widehat{\phi}_\pi$. Every cutoff in this region yields the lender the same surplus π. In particular, the lender can choose the lower support point $\underline{\mu}_\pi$ as a best response. Since the canonical demand puts no mass below this point, every realized posterior then accepts financing. Thus the demand corresponding to $\widehat{\phi}_\pi$ admits an efficient equilibrium that preserves the original lender surplus with a borrower net surplus of $\kappa = \Delta - \pi$.

Moreover, Corollary 1 implies that if $\pi' < \pi$ then $\widehat{F}_\pi \preceq_{\text{cx}} \widehat{F}_{\pi'}$. Thus $\widehat{\phi}_\pi$ becomes more informative as the lender surplus falls. To see this let $\pi' < \pi$ and mix the canonical demand $\widehat{F}_{\pi'}$ with no information,

$$F_\lambda = (1-\lambda)\widehat{F}_{\pi'} + \lambda\delta_m.$$

The mixture remains feasible. As in the proof of Theorem 1, lender payoff is affine in λ at every cutoff, so the optimized lender payoff is continuous in λ. By the intermediate value theorem, there exists some λ that implements π and $\widehat{\phi}_\pi \le \phi_{F_\lambda} \le \widehat{\phi}_{\pi'}$. Therefore, $\widehat{F}_\pi \preceq_{\text{cx}} \widehat{F}_{\pi'}$ and

$$[\underline{\mu}_\pi\,,\, \overline{\mu}_\pi] \subseteq [\underline{\mu}_{\pi'}\,,\, \overline{\mu}_{\pi'}]$$

We now use these observations to derive a comparative static between the selection slope D and the set of efficient borrower–lender surplus shares.

Comparative Statics

In this section we study how changes to the strength of adverse selection (steepness of D) effect the set of efficient equilibrium outcomes. Recall that, when type μ is marginal, the lender's payoff from financing type $s \ge \mu$ is

$$\Delta - (s-\mu)D(\mu).$$

Thus $D(\mu)$ measures how quickly the lender's value decreases (or equivalently, the rate at which the borrower accumulates rents) as the composition of accepting borrowers shifts toward riskier posterior types.

Consider two markets A and B with the same prior and total gains from financing, and let D^A and D^B denote the map from the marginal type to the slope of borrower's indirect utility in markets A and B respectively. We say that A has *stronger selection* than B if

$$D^A(s) \geq D^B(s) \qquad \text{for every } s \in [0,1]. \tag{8}$$

For $j \in \{A, B\}$, let $\mathcal{W}^j$ denote the set of borrower–lender surplus pairs that can be implemented in an efficient equilibrium, and define the lowest lender surplus the borrower can induce by

$$\pi_j^* := \inf_{F \in \mathcal{F}(m)} \max_{s \in [0,1]} \Pi_F^j(s). \tag{9}$$

Proposition 1. *Suppose Assumption 1 holds in both markets. If A has stronger selection than B, then*

$$\pi_A^* \leq \pi_B^* \qquad \textit{and} \qquad \mathcal{W}^B \subseteq \mathcal{W}^A.$$

Proof. For any feasible demand F and cutoff s, Equation 6 and Equation 8 imply

$$\Pi_F^A(s) \leq \Pi_F^B(s).$$

Taking the maximum over s and then the infimum over F gives $\pi_A^* \leq \pi_B^*$.

Now take any $(\kappa, \pi) \in \mathcal{W}^B$ and let F implement it. Evaluating the same demand in market A gives an equilibrium lender surplus

$$\pi' := \max_s \Pi_F^A(s) \leq \pi.$$

By Theorem 1, π' has an efficient implementation in market A. We can construct a lottery of this experiment and an uninformative experiment which can implement any payoff between π' and Δ. Applying Theorem 1 once more gives an efficient implementation, $\widehat{\phi}_\pi$, of π in market A. Thus the surplus pair (κ, π) is in $\mathcal{W}^A$. □

The borrower's experiment changes the posterior composition of borrowers willing to accept each contract, and the lender changes the contract in response. When D is larger, a shift toward riskier borrowers reduces the lender's payoff by more. Private learning can then support lower lender surplus and more efficient outcomes. Because total gains from financing remain Δ, stronger selection redistributes surplus in favor of the borrower.

Now we will describe risk dispersion and curvature of the repayment as two primitive ways of changing the this selection margin.

Risk Dispersion

We first strengthen selection by increasing the separation in risk between the two project states. Consider a second pair of return distributions $\widetilde{G}_0, \widetilde{G}_1$ satisfying

$$\widetilde{G}_0 \preceq_{\text{cx}} G_0 \preceq_{\text{cx}} G_1 \preceq_{\text{cx}} \widetilde{G}_1, \tag{10}$$

where all four distributions have mean $\overline{R}$. Thus expected returns and total gains from financing are unchanged.

Let $\widetilde{u}_\theta$, $\widetilde{\hat{r}}$, and $\widetilde{D}$ denote the corresponding objects in the more dispersed economy.

Lemma 1. *Suppose Assumption 1 holds. Then*

$$\widetilde{D}(\mu) \geq D(\mu) \qquad \textit{for every } \mu \in (0,1).$$

Proof. Because the borrower residual claim is convex in R, Equation 10 implies

$$\widetilde{u}_0(r) \leq u_0(r) \quad \text{and} \quad \widetilde{u}_1(r) \geq u_1(r)$$

for every r. Suppose instead that $\widetilde{D}(\mu) < D(\mu)$. The reservation condition gives

$$u_0(\hat{r}(\mu)) = R_0 - \mu D(\mu), \quad \text{and} \quad u_1(\hat{r}(\mu)) = R_0 + (1-\mu)D(\mu),$$

and analogously in the more dispersed economy. The state-0 comparison implies $\widetilde{\hat{r}}(\mu) < \hat{r}(\mu)$, while the state-1 comparison implies $\widetilde{\hat{r}}(\mu) > \hat{r}(\mu)$, a contradiction. □

Corollary 2. *Under Equation 10,*

$$\widetilde{\pi}^* \leq \pi^*,$$

and every efficient borrower–lender surplus pair implementable in the original market remains implementable in the more dispersed market.

Proof. This follows immediately from Lemma 1 and Proposition 1. □

Greater risk dispersion therefore strengthens the selection force leading to steeper D. A riskier borrower becomes relatively more willing to accept financing and, under a concave repayment claim, relatively less valuable to the lender. By Proposition 1, private learning has greater redistributive force even though expected project returns and total surplus are unchanged.

Security Curvature

We next hold project risk fixed and change the curvature of the repayment claim. Consider two repayment families T^A and T^B satisfying Assumption 1. We say that A is *more selection-concave* than B if, for every r,

$$T^A(\cdot, r) - T^B(\cdot, r) \quad \text{is concave,} \tag{11}$$

and the two contracts have the same expected repayment in the safe state,

$$\mathbb{E}_{G_0}\left[T^A(R,r)\right] = \mathbb{E}_{G_0}\left[T^B(R,r)\right]. \tag{12}$$

This normalization holds fixed the value of the claim on the safe project, while the curvature condition changes how repayment responds to project risk.

Lemma 2. *If A is more selection-concave than B, then*

$$D^A(\mu) \geq D^B(\mu) \qquad \textit{for every } \mu \in (0,1).$$

Proof. The Equation 1, Equation 11, and Equation 12 imply

$$u_0^A(r) = u_0^B(r), \qquad u_1^A(r) \geq u_1^B(r)$$

for every r. Hence $\hat{r}^A(\mu) \geq \hat{r}^B(\mu)$. Since the state-0 utility is common across the two families and decreasing in r,

$$D^j(\mu) = \frac{R_0 - u_0^j(\hat{r}^j(\mu))}{\mu}, \qquad j \in \{A, B\},$$

which gives the result. □

Corollary 3. *If A is more selection-concave than B, then*

$$\pi_A^* \leq \pi_B^*,$$

and every efficient borrower–lender surplus pair implementable under market B remains implementable under market A.

Proof. This follows immediately from Lemma 2 and Proposition 1. □

With equal mean returns, an affine repayment claim does not distinguish the two project states in expected repayment and generates no risk-based selection. Greater concavity makes the lender's value from repayment more sensitive to project risk, increases D, and strengthens the redistributive effect of borrower's private learning.

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A Proof of Theorem 1

Proof of Theorem 1. Since π is implementable, Φ_π is nonempty and the pointwise lower envelope $\phi_\pi^{\inf}$ is well defined. Let $\widehat{\phi}_\pi$ be its convex minorant.

Feasibility. Let

$$\phi^0(s) := (m-s)^+$$

be the call function of the uninformative posterior distribution. We have $\phi^0 \leq \phi$ for every feasible call function ϕ, hence $\phi^0 \leq \phi_\pi^{\inf}$. Since ϕ^0 is convex we get

$$\phi^0 \leq \widehat{\phi}_\pi \leq \phi_\pi^{\inf}.$$

All feasible call functions satisfy $\phi(0) = m$ and $\phi(1) = 0$, so the inequalities above imply that $\widehat{\phi}_\pi(0) = m$ and $\widehat{\phi}_\pi(1) = 0$. Moreover, $\widehat{\phi}_\pi$ is nonnegative, convex, and non-increasing (the convex minorant of a non-increasing function is non-increasing). Because $\widehat{\phi}_\pi$ touches ϕ^0 at zero, its right slope at zero is at least -1; convexity then implies that every slope lies in $[-1, 0]$. Thus $\widehat{\phi}_\pi$ is a feasible call function and corresponds to a unique Bayes-plausible posterior distribution $\widehat{F}_\pi$.

The lender-profit ceiling. For a feasible call function ϕ, write

$$\Pi_\phi(s) := \Delta[-\phi'_-(s)] - D(s)\phi(s).$$

Every $\phi \in \Phi_\pi$ satisfies $\Pi_\phi(s) \leq \pi$ for every s.

Consider first a contact point $s \in (0, 1]$ at which $\widehat{\phi}_\pi(s) = \phi_\pi^{\inf}(s)$. Choose $\phi_n \in \Phi_\pi$ with $\phi_n(s) \to \widehat{\phi}_\pi(s)$. For any $t < s$, convexity gives

$$-(\widehat{\phi}_\pi)'_-(s) \leq \frac{\widehat{\phi}_\pi(t) - \widehat{\phi}_\pi(s)}{s-t} \leq -\phi'_{n,-}(t) + \frac{\phi_n(s) - \widehat{\phi}_\pi(s)}{s-t}.$$

Using the lender-profit ceiling for ϕ_n at t, and the fact that every feasible call function is one-Lipschitz,

$$\begin{aligned}\Delta[-(\widehat{\phi}_\pi)'_-(s)] &\leq \pi + D(t)\phi_n(t) + \Delta\frac{\phi_n(s) - \widehat{\phi}_\pi(s)}{s-t} \\ &\leq \pi + D(t)\left[\phi_n(s) + (s-t)\right] + \Delta\frac{\phi_n(s) - \widehat{\phi}_\pi(s)}{s-t}.\end{aligned}$$

First let $n \to \infty$, and then let $t \uparrow s$. Continuity of D yields

$$\Delta[-(\widehat{\phi}_\pi)'_-(s)] - D(s)\widehat{\phi}_\pi(s) \leq \pi.$$

At $s = 0$ the same inequality follows directly because all feasible call functions have the same gross and selection contribution at $\hat{r}(0)$.

As $\widehat{\phi}_\pi$ is a convex minorant of $\phi_\pi^{\inf}(s)$, if s is a type such that $\widehat{\phi}_\pi(s) < \phi_\pi^{\inf}(s)$ then there is an interval (a, b) containing s such $(a, b) \subseteq \{s : \widehat{\phi}_\pi(s) < \phi_\pi^{\inf}(s)\}$ and $\widehat{\phi}_\pi(b) = \phi_\pi^{\inf}(b)$. The distribution $\widehat{F}_\pi$ therefore has no mass on (a, b). The set of borrowers accepting $\hat{r}(\cdot)$ is then unchanged as s moves from any point in (a, b) to b, while the contract terms become more demanding. Lender repayment is non-decreasing in the contract term, so for every μin $[0, 1]$

$$\Pi_{\widehat{\phi}_\pi}(\mu) \leq \Pi_{\widehat{\phi}_\pi}(b) \leq \pi. \tag{13}$$

Exact implementation. Define the following

$$\mathcal{C}_\pi := \left\{\phi :\ \phi \text{ is feasible and } \Pi_\phi(s) \leq \pi \text{ for every } s\right\}.$$

We first show that

$$\widehat{\phi}_\pi \leq \phi \qquad \text{for every } \phi \in \mathcal{C}_\pi. \tag{14}$$

Take any $\phi \in \mathcal{C}_\pi$ and mix its posterior distribution with the uninformative one. The resulting call function is

$$\phi_\lambda = (1 - \lambda)\phi + \lambda\phi^0.$$

Since $\phi^0 \leq \phi$, we have $\phi_\lambda \leq \phi$. Let

$$V(\lambda) := \max_s \Pi_{\phi_\lambda}(s).$$

For each cutoff the profit is affine in λ. Moreover,

$$|V(\lambda) - V(\lambda')| \leq |\lambda - \lambda'| \sup_s \left|\Pi_{\phi^0}(s) - \Pi_\phi(s)\right|,$$

so V is Lipschitz continuous. The lender surplus against an uninformative experiment is $V(1) = \Delta \geq \pi$. Hence by intermediate value theorem, there exists some λ^*, $V(\lambda^*) = \pi$, so $\phi_{\lambda^*} \in \Phi_\pi$. By construction of $\phi_\pi^{\inf}$ and $\widehat{\phi}_\pi$,

$$\widehat{\phi}_\pi \leq \phi_\pi^{\inf} \leq \phi_{\lambda^*} \leq \phi,$$

which proves Equation 14.

Now Equation 13 gives $\widehat{\phi}_\pi \in \mathcal{C}_\pi$. Suppose, toward a contradiction, that

$$\max_s \Pi_{\widehat{\phi}_\pi}(s) < \pi.$$

Mix $\widehat{\phi}_\pi$ with the uninformative call function:

$$\phi_\lambda = (1-\lambda)\widehat{\phi}_\pi + \lambda\phi^0.$$

At $\lambda = 0$ optimized lender profit is strictly below π, while at $\lambda = 1$ it equals Δ and hence is at least π. Continuity therefore gives some $\lambda^* > 0$ for which $\phi_{\lambda^*} \in \Phi_\pi$. Since $\phi^0 \leq \widehat{\phi}_\pi$ we get by construction

$$\phi_{\lambda^*} \leq \widehat{\phi}_\pi,$$

But from the argument above, ϕ_{λ^*} in Φ implies that $\widehat{\phi}_\pi \leq \phi_{\lambda^*}$. Thus $\widehat{\phi}_\pi = \phi^0$ which contradicts $\max_s \Pi_{\widehat{\phi}_\pi}(s) < \pi$. Therefore

$$\max_s \Pi_{\widehat{\phi}_\pi}(s) = \pi,$$

so $\widehat{\phi}_\pi$ itself implements π.

Finally, for every $\phi \in \Phi_\pi$,

$$\widehat{\phi}_\pi \leq \phi_\pi^{\inf} \leq \phi.$$

All such call functions have the same mean m. The call-function characterization of convex order therefore gives

$$\widehat{F}_\pi \preceq_{\mathrm{cx}} F$$

for every posterior distribution F implementing π. This proves least informativeness.

$\square$