Supplemental Appendix for
Employer Competition and Certification

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This supplementary appendix characterizes the structure of the optimal test allocation mechanism under assumption 1 of the main text and introduces an extension of the main model without soft information.

1 Optimal Test Design

The next Proposition characterizes optimal tests under assumption 1.

Proposition 1. *If assumption 1 holds and $0 \leq s_b \leq s_t \leq 1$, then the optimal test takes one of the following form for some $0 \leq \mu_1 \leq \mu_2 \leq \mu_2 \leq \mu_3 \leq \mu_4 \leq 1$.*

$$\rho(\mu) = \begin{cases} (0,0,0,0) & \mu < \mu_0 \\ (0,1,0,1) & \mu \in [\mu_0,\mu_1) \\ (0,1,u,0) & \mu \in [\mu_1,\mu_2) \\ (1,0,1,0) & \mu \in [\mu_2,\mu_3) \\ (1,0,0,0) & \mu \in [\mu_3,1] \end{cases} \quad \text{or} \quad \begin{cases} (0,0,0,0) & \mu < \mu_0 \\ (0,1,0,1) & \mu \in [\mu_0,\mu_1) \\ (0,1,u,0) & \mu \in [\mu_1,\mu_2) \\ (0,1,0,0) & \mu \in [\mu_2,\mu_3) \\ (1,0,0,0) & \mu \in [\mu_3,1] \end{cases} \quad \text{or} \quad \begin{cases} (0,0,0,0) & \mu < \mu_0 \\ (0,1,0,1) & \mu \in [\mu_0,\mu_1) \\ (u,0,0,1) & \mu \in [\mu_1,\mu_2) \\ (1,0,1,0) & \mu \in [\mu_2,\mu_3) \\ (1,0,0,1) & \mu \in [\mu_3,\mu_4) \\ (1,0,0,0) & \mu \in [\mu_4,1] \end{cases}$$

Moreover, if $\frac{1-F(\mu)}{f(\mu)}$ is convex then the test can be simplified further to

$$\rho(\mu) = \begin{cases} (0,0,0,0) & \mu < \mu_0 \\ (0,1,0,1) & \mu \in [\mu_0,\mu_1) \\ (0,1,u,0) & \mu \in [\mu_1,\mu_2) \\ (1,0,1,0) & \mu \in [\mu_2,\mu_3) \\ (1,0,0,0) & \mu \in [\mu_3,1] \end{cases} \quad \text{or} \quad \begin{cases} (0,0,0,0) & \mu < \mu_0 \\ (0,1,0,1) & \mu \in [\mu_0,\mu_1) \\ (0,1,u,0) & \mu \in [\mu_1,\mu_2) \\ (0,1,0,0) & \mu \in [\mu_2,\mu_3) \\ (1,0,0,0) & \mu \in [\mu_3,1] \end{cases} \quad \text{or} \quad \begin{cases} (0,0,0,0) & \mu < \mu_0 \\ (0,1,0,1) & \mu \in [\mu_0,\mu_1) \\ (1,0,1,0) & \mu \in [\mu_1,\mu_2) \\ (1,0,0,1) & \mu \in [\mu_2,\mu_3) \\ (1,0,0,0) & \mu \in [\mu_3,1] \end{cases}$$

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Proof. Part of the Proposition is established in section A.7 of the main text for proving claim 2. The proof of the proposition then follows from noting that the following pairs of test allocation cannot occur together:

1. $(0,1,0,0)$ and $(1,0,1,0)$: To prove that these allocations do not occur together, I proceed by contradiction and construct profitable perturbations for the certifier (the latter parts are dealt similarly). By monotonicity of K, the allocation $(1,0,1,0)$ is offered to a lower interval of types than $(0,1,0,0)$. Let $(1,0,1,0)$ be offered to types in I_1 and $(0,1,0,0)$ be offered to types in I_2, where $I_1 < I_2$. A profitable perturbation exists by choosing subsets $I_1' \subset I_1$ and $I_2' \subset I_2$ such that $\int_{I_1'} \mu \, dF(\mu) = u \int_{I_2'} \mu \, dF(\mu)$. Construct the perturbation by first setting $\rho(I_1') = (0,1,u,0)$ and $(\ \rho_h^t(I_2'), \rho_h^b(I_2')\ ) = (1,0)$. Choose $J \subset I_2$ such that $(1-u)\int_{I_1'}(1-\mu)\, dF(\mu) = \int_J (1-\mu)\, dF(\mu)$ Now, perturb the new allocation in J by setting $\rho_l(J) = (u,0)$. This preserves the gross utility but flattens the indirect utility, improving revenue.

2. $(0,1,0,0)$ and $(1,0,0,1)$: If $u < 1/2$ then by monotonicity of K, the allocation $(0,1,0,0)$ is offered to a lower interval of types than $(1,0,1,0)$. Let $(0,1,0,0)$ be offered to types in I_1 and $(1,0,1,0)$ be offered to types in I_2, where $I_1 < I_2$. A profitable perturbation exists by choosing subsets $I_1' \subset I_1$ and $I_2' \subset I_2$ such that $\int_{I_1'}(1-\mu)\, dF(\mu) = \int_{I_2'}(1-\mu)\, dF(\mu)$. Construct the perturbation by first setting $\rho_l(I_1') = (0,1)$ and $\rho_l(I_2') = (0,0)$. This improves revenue by calculations in the remark in section **??**.
   If $u \geq 1/2$ then by monotonicity of K, the allocation $(1,0,1,0)$ is offered to a lower interval of types than $(0,1,0,0)$. Let $(1,0,1,0)$ be offered to types in I_1 and $(0,1,0,0)$ be offered to types in I_2, where $I_1 < I_2$. A profitable perturbation exists by choosing subsets $I_1' \subset I_1$ and $I_2' \subset I_2$ such that $\int_{I_1'} \mu \, dF(\mu) = \int_{I_2'} \mu \, dF(\mu)$. Construct the perturbation by first setting $\rho_h(I_1') = (0,1)$ and $\rho_h(I_2') = (1,0)$. This improves revenue by calculations at the end of section **??**.

3. $(0,1,u,0)$ and $(1,0,0,1)$: By monotonicity of K, the allocation $(0,1,u,0)$ is offered to a lower interval of types than $(1,0,0,1)$. Let $(0,1,u,0)$ be offered to types in I_1 and $(1,0,0,1)$ be offered to types in I_2, where $I_1 < I_2$. A profitable perturbation exists by choosing subsets $I_1' \subset I_1$ and $I_2' \subset I_2$ such that $\int_{I_1'}(1-\mu)\, dF(\mu) = \int_{I_2'}(1-\mu)\, dF(\mu)$. Construct the perturbation by first setting $\rho_l(I_1') = (0,1)$ and $\rho_l(I_2') = (u,0)$. Partition I_2' into J and J' such that $J < J'$ and $u\int_{I_2'}(1-\mu)\, dF(\mu) = \int_{J'}(1-\mu)\, dF(\mu)$. Now, perturb the new allocation on I_2' by setting $\rho_l(J) = (1,0)$ and $\rho_l^t(J') = (0,0)$. This improves revenue by calculations in the remark in section **??**.

4. $(u,0,0,1)$ and $(0,1,0,0)$: By monotonicity of K, the allocation $(u,0,0,1)$ is offered to a lower interval of types than $(0,1,0,0)$. Let $(u,0,0,1)$ be offered to types in I_1 and $(0,1,0,0)$ be offered to types in I_2, where $I_1 < I_2$. A profitable perturbation exists by choosing subsets $I_1' \subset I_1$ and $I_2' \subset I_2$ such that $\int_{I_1'} \mu \, dF(\mu) = \int_{I_2'} \mu \, dF(\mu)$. Construct the perturbation by first

setting $\rho_h(I_1') = (0,1)$ and $\rho_h(I_2') = (u,0)$. Partition I_2' into J and J' such that $J < J'$ and $u \int_{I_2'} \mu \, dF(\mu) = \int_{J'} \mu \, dF(\mu)$. Now, perturb the new allocation on I_2' by setting $\rho_h(J) = (0,0)$ and $\rho_h^t(J') = (1,0)$. This improves revenue by calculations at the end of section **??**.

The second statement follows from considering the allocation

$$\rho(\mu) = \begin{cases} (0,0,0,0) & \mu < \mu_0 \\ (0,1,0,1) & \mu \in [\mu_0, \mu_1) \\ (u,0,0,1) & \mu \in [\mu_1, \mu_2) \\ (1,0,1,0) & \mu \in [\mu_2, \mu_3) \\ (1,0,0,1) & \mu \in [\mu_3, \mu_4) \\ (1,0,0,0) & \mu \in [\mu_4, 1] \end{cases}$$

For ρ to be an optimal solution to the relaxed problem, it must be that $\mu - \frac{1-F(\mu)}{f(\mu)} \leq 0$ for $\mu \in [\mu_1, \mu_2)$. Also $\mathbf{E}[\ \mu \mid a_b\ ] \leq \mu_2$.
If $\frac{1-F(\mu)}{f(\mu)}$ is convex, then $\mu_2 \leq \mathbf{E}[\mu]$, see section A.6 of the main text for details. As $\mathbf{E}[\mu] < \frac{-v_l}{v_h - v_l}$, we get $\mathbf{E}[\ \mu \mid a_b\ ] < \frac{-v_l}{v_h - v_l}$, a contradiction. □

Proposition 2: The set of equilibrium (in pure strategy) standards is

$$\mathcal{E} = \left\{ (s_t, s_b) = (\ s,\ us\ ) \ \middle|\ s \in \left[\min\left\{ \frac{1}{u} \frac{-v_l}{v_h - v_l}, 1 \right\}, 1 \right] \right\}$$

1.1 No Soft Information

In this extension, the employers can observe the applicant's type. This shuts down the soft information channel. I provide an overview of the qualitative difference of this assumption.[1]

Utilizing the revelation principle, we can again focus on incentive compatible, obedient, and individually rational direct mechanisms. The main difference, compared to section 3 in the main text, is in how the revelation principle is invoked. Instead of restricting the messages $\mathcal{M}_E = \Delta(A)$, the mechanism announces the applicant's reported type along with a firm-specific hiring recommendation. More precisely, $\mathcal{M}_E = \Delta(A) \times [0,1]$.

For an incentive compatible direct mechanism, the applicant's reported type is his true type. As the employers are privy to the applicant's true type μ, requiring the mechanism to announce the applicant's reported type does not restrict equilibrium implementation. This simplifies the analysis by disciplining the employer's response to a message obtained through a misreport by the applicant. I elaborate on this later.

[1]The design of an optimal mechanism poses challenges tangential to the main model, stemming from failure of revenue equivalence. See Ely (2025), Celik and Strausz (2025), and Mäkimattila et al. (2025) for further discussion about settings similar this extension.

Another distinction from section 3 is that the employer hires the applicant if and only if the true type and reported type are the same and the certifier recommends hiring. This leads to the following obedience constraint that hold type-by-type for all $\mu \in [0,1]$

$$\mu(1-s_t)\rho_h^t(\mu) - s_t(1-\mu)\rho_l^t(\mu) \geq 0 \qquad \text{(Pointwise Obedience)}$$
$$\mu(1-s_b)\rho_h^b(\mu) - s_b(1-\mu)\rho_l^b(\mu) \geq 0$$
$$\mu(1-s_t)\rho_h^b(\mu) - s_t(1-\mu)\rho_l^b(\mu) < 0$$

Benchmark Without Soft Information: First, I consider the variation of section 4 of the main text without soft information. As expected, without screening frictions, the absence of soft information does not affect employers' competitiveness and features no exclusion.

Proposition 2. *The set of equilibrium standards (in pure strategy) is*

$$\mathcal{E} = \left\{ (s_t, s_b) = \ (\ s, \ us \ ) \ \middle| \ s \in \left[\min \left\{ \frac{1}{u} \frac{-v_l}{v_h - v_l}, 1 \right\}, 1 \right] \ \right\}$$

Proof. Fix some standards (s_t, s_b), recall the designer's problem in section **??** is

$$\max \ \int_0^1 V(\mu) \, dF(\mu)$$

Where the ρ satisfies (Pointwise Obedience). We get that the optimal test allocation is the following

If $u \leq \frac{s_b}{s_t}$ then the optimal test has the following form

$$\begin{pmatrix} \rho_h^t(\mu) & \rho_h^b(\mu) \\ \rho_l^t(\mu) & \rho_l^b(\mu) \end{pmatrix} := \begin{cases} \begin{pmatrix} 1 & 0 \\ \frac{\mu(1-s_t)}{s_i(1-\mu)} & 0 \end{pmatrix} & \mu < s_t \\ \\ \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix} & \mu \geq s_t \end{cases}$$

The corresponding employer surplus is given by

$$U_t(\ s_t, s_b) = \int_0^1 \mu \, v_h \, dF(\mu) + \int_0^{s_t} \mu \, \frac{(1-s_t)}{s_t} \, v_l \, dF(\mu) + \int_{s_t}^1 (1-\mu) \, v_l \, dF(\mu)$$
$$U_b(\ s_t, s_b) = 0$$

U_t is increasing in s_t.

If $u \geq \frac{s_b}{s_t}$ then the optimal test has the following form

$$\begin{pmatrix} \rho_h^t(\mu) & \rho_h^b(\mu) \\ \rho_l^t(\mu) & \rho_l^b(\mu) \end{pmatrix} := \begin{cases} \begin{pmatrix} 0 & 1 \\ 0 & \frac{\mu(1-s_b)}{s_b(1-\mu)} \end{pmatrix} & \mu < s_b \\ \begin{pmatrix} \frac{(\mu-s_b)s_t}{(s_t-s_b)\mu} & \frac{(s_t-\mu)s_b}{(s_t-s_b)\mu} \\ \frac{(\mu-s_b)(1-s_t)}{(s_t-s_b)(1-\mu)} & \frac{(s_t-\mu)(1-s_b)}{(s_t-s_b)(1-\mu)} \end{pmatrix} & \mu \in [s_b, s_t) \\ \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix} & \mu \geq s_t \end{cases}$$

And employer surplus is given by

$$U_t(\, s_t, s_b) = \int_{s_t}^{1} [\, \mu \, v_h + (1-\mu) \, v_l \,] \; dF(\mu) + \int_{s_b}^{s_t} (\mu - s_b) \left[\frac{v_h s_t + v_l(1-s_t)}{s_t - s_b} \right] \; dF(\mu)$$

$$U_b(\, s_t, s_b) = \int_{0}^{s_b} \left[\mu \, v_h + \mu \, v_l \, \frac{1-s_b}{s_b} \right] \; dF(\mu) + \int_{s_b}^{s_t} (s_t - \mu) \left[\frac{v_h s_b + v_l(1-s_b)}{s_t - s_b} \right] \; dF(\mu)$$

In either case, the employers have a positive payoff only if $s_t \geq \frac{-v_l}{v_h - v_l}$ and $s_b \geq \frac{-v_l}{v_h - v_l}$ respectively. This gives a lower bound on the hiring standard in any equilibrium.
The derivative of U_t with respect to s_t is

$$\frac{\partial}{\partial s_t} U_t(\, s_t, s_b) = \int_{s_b}^{s_t} (\mu - s_b) \frac{-v_l + (v_l - v_h)s_b}{(s_t - s_b)^2} \; dF(\mu)$$

This is negative as $s_b \geq \frac{-v_l}{v_h - v_l}$. So U_t is maximized at $s_t = s_b/u$.
The derivative of U_b with respect to s_b is

$$\frac{\partial}{\partial s_b} U_b(\, s_t, s_b) = \int_{s_b}^{s_t} (s_t - \mu) \frac{(v_h - v_l)s_t + v_l}{(s_t - s_b)^2} \; dF(\mu)$$

This is positive as $s_t \geq \frac{-v_l}{v_h - v_l}$. So U_b is maximized at $s_b = us_t$.
The set of equilibrium (in pure strategy) standards is

$$\mathcal{E} = \left\{ \; (\, s, \; us \,) \; \middle| \; s \in \left[\min \left\{ \frac{1}{u} \frac{-v_l}{v_h - v_l}, 1 \right\}, 1 \right] \; \right\}$$

□

Remark. Given some hiring standards (s_t, s_b). As types are observable to the certifier and the employers, each mechanism can be regarded as a function from the applicant's type to a distribution over posterior expectation of θ.

If $us_t < s_b$, then the certifier allocates types $\mu \in [0, s_t]$ to posteriors 0 and s_t. If $us_t > s_b$, then the certifier allocates types $\mu \in [0, s_b]$ to posteriors 0 and posterior s_b, type $\mu \in [s_b, s_t]$ to posterior s_b and s_t. In either case, all types above s_t are left uninformed, hence type $\mu \in [s_t, 1]$ is assigned a posterior μ. By Bayes-plausibility, the expected posterior belief induced by the mechanism for type μ applicant is his prior μ. Importantly, conditional on $\theta = h$, the applicant is always hired.

In equilibrium, any hired applicant must have a positive expected payoff for either employer. Thus, the employers choose hiring standards $s_t,\ s_b \geq \frac{-v_l}{v_h - v_l}$. In particular, if $u < \frac{-v_l}{v_h - v_l}$ then only the top employer hires in equilibrium. In this case $s_t = 1$.

If $u > \frac{-v_l}{v_h - v_l}$, then the bottom employer can undercut the top employer if the top employer chooses $s_t = 1$. Keeping the bottom employer's standard fixed, the top employer is willing to lower its standards till $s_t = s_b/u$. This allows the top employer to peel off applicants, with positive expected value, away from the bottom employer. Keeping the top employer's standard fixed, the bottom employer employer is willing to increase its standards till $s_b = us_t$. This allows the bottom employer to poach high ability applicants away from the top employer, while reducing the chance of hiring a low ability applicant.

Thus, in equilibrium, the employers choose hiring standards that make the certifier indifferent between allocating the applicants to the bottom employer with a positive probability and only allocating them to the top employer.

Informed Employers, Uninformed Certifier: Now I consider the analogue to the main model in which the employer observes the applicant's type, but the certifier does not. As in section 5 of the main text, the distortions from price discrimination affect the employer competitiveness.

Recall the mechanism announces an employer-specific hiring recommendation and the applicant's reported type. In equilibrium, the employer knows the test that the applicant self-selects into. Without loss, we can define the type μ applicant's utility from a test ρ as if the employers can observe the test chosen.

The employers' posterior belief about a type μ applicant after observing message a_j and test ρ is given by

$$\eta^j(\mu, \rho) := \frac{\mu \rho_h(a_j)}{\mu \rho_h(a_j) + (1-\mu)\rho_l(a_j)}$$

Where $j \in \{t, b\}$. By convention let $\eta^j(\mu, \rho) = 0$ if $\mu \rho_h(a_j) + (1-\mu)\rho_l(a_j) = 0$.

Given hiring standards $s_t \geq s_b$, the applicant's gross utility from a test ρ and type μ is given by

$$V(\mu, \rho) = \mathbf{E}_\mu \left[\rho^t_\theta(\mu) \cdot \left(1_{\eta^t(\mu,\rho) \geq s_t} + u 1_{s_t > \eta^t(\mu,\rho) \geq s_b} \right) \; + \; \rho^b_\theta(\mu) \cdot \left(1_{\eta^b(\mu,\rho) \geq s_t} + \; u 1_{s_t > \eta^b(\mu,\rho) \geq s_b} \right) \right]$$

Given a direct mechanism (ρ, φ) let $\eta^j(\mu, \nu), := \eta^j(\mu, \rho(\nu))$, $\eta^j(\mu) := \eta^j(\mu, \mu)$ and $V(\mu, \nu) := V(\mu, \rho(\nu))$.

Incentive compatibility requires that for all types μ, ν we have

$$V(\mu,\nu) - \varphi(\nu) \leq V(\mu) - \varphi(\mu) =: \mathcal{U}(\mu)$$

As testing is voluntary, individual rationality requires the following

$$\mathcal{U}(\mu) \geq \begin{cases} 1 & \text{if} \quad \mu \geq s_t \\ u & \text{if} \quad \mu \geq s_b \\ 0 & \text{otherwise} \end{cases}$$

The applicant's outside option is discontinuous in the applicant's type. As type $\mu = 0$ is never hired we must have $\mathcal{U}(0) = 0$.

The applicant's payoff is not linear in his type because of the discontinuity in payoffs and outside options. Thus, the envelope representation of incentive compatible mechanisms from the main text does not hold.[2] Given these differences, I leave further analysis of this setting for future work.

References

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[2]See Proposition 2 in Rochet (1987). Also see Carbajal and Ely (2013) and Kos and Messner (2013) for general screening problems with non-linear payoffs.